① 141, ② 3100, ③ $x=-2$, $y=4$, ④ $x=\frac{73}{31}$, $y=\frac{2}{31}$

⑤ $x=-1$, $y=0$, $z=2$ ⑥ $x=\frac{89}{42}$, $y=-\frac{16}{21}$, $z=-\frac{83}{42}$

⑤
$$\begin{aligned} -3x \quad\quad\; + z &= 5 \\ x - 3y + 4z &= 7 \\ 6y - 2z &= -4 \end{aligned}$$

$$x = \frac{\begin{vmatrix} 5 & 0 & 1 \\ 7 & -3 & 4 \\ -4 & 6 & -2 \end{vmatrix}}{\begin{vmatrix} -3 & 0 & 1 \\ 1 & -3 & 4 \\ 0 & 6 & -2 \end{vmatrix}} = \frac{-60}{60} = -1$$

$$\left|\begin{matrix} 5 & 0 & 1 \\ 7 & -3 & 4 \\ -4 & 6 & 2 \end{matrix}\right| \begin{matrix} 5 & 0 \\ 7 & -3 \\ -4 & 6 \end{matrix}$$

$$= -30 + 0 + 42 - 12 - 120 - 0 = -60$$

$$\left|\begin{matrix} -3 & 0 & 1 \\ 1 & -3 & 4 \\ 0 & 6 & -2 \end{matrix}\right| \begin{matrix} -3 & 0 \\ 1 & -3 \\ 0 & 6 \end{matrix}$$

$$= -18 + 0 + 6 - 0 - (-72) - 0 = 60$$

$$y = \frac{\begin{vmatrix} -3 & 5 & 1 \\ 1 & 7 & 4 \\ 0 & -4 & -2 \end{vmatrix}}{60} = \frac{0}{60} = 0$$

$$\begin{vmatrix} -3 & 5 & 1 \\ 1 & 7 & 4 \\ 0 & -4 & -2 \end{vmatrix} \begin{matrix} -3 & 5 \\ 1 & 7 \\ 0 & -4 \end{matrix}$$

$$= 42 + 0 + (-4) - 0 - 48 - (-10) = 0$$

$$z = \frac{\begin{vmatrix} -3 & 0 & 5 \\ 1 & -3 & 7 \\ 0 & 6 & -4 \end{vmatrix}}{60} = \frac{120}{60} = 2$$

$$\begin{vmatrix} -3 & 0 & 5 \\ 1 & -3 & 7 \\ 0 & 6 & -4 \end{vmatrix} \begin{matrix} -3 & 0 \\ 1 & -3 \\ 0 & 6 \end{matrix}$$

$$= -36 + 0 + 30 - 0 - (-126) - 0 = 120$$

6

$$y = \frac{\begin{vmatrix} 2 & 5 & 0 \\ 0 & -7 & 2 \\ 9 & 11 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & 0 \\ 0 & 4 & 2 \\ 9 & 8 & 1 \end{vmatrix}} = \frac{32}{-42} = -\frac{16}{21}$$

$$\begin{vmatrix} 2 & -1 & 0 \\ 0 & 4 & 2 \\ 9 & 8 & 1 \end{vmatrix} \begin{matrix} 2 & -1 \\ 0 & 4 \\ 9 & 8 \end{matrix}$$

$$= 8 + (-18) + 0 - 0 - 32 - 0 = -42$$

Complex Numbers

Ex: Solve $x^2+1=0$.

$$x^2+1-1=0-1$$
$$x^2=-1$$

There is no real number whose square is negative.

We invent a solution which we will call j.

Optional: use i instead of j.

$$x^2=-1$$

j is a solution

$$\boxed{j^2=-1} \qquad j\cdot j=-1$$

In essence, $j=\sqrt{-1}$

j is called a complex number.

Other complex numbers: $2j$, $-1+j$, $j+j=2j$

$-3+2j$, $8-7j$, $-16+\frac{3}{4}j$

Rectangular Form → $-3+2j$

The complex part

The real part

A real number

$$\underbrace{-4 = -4 + 0j}_{\text{A complex number}}$$

Real numbers are also complex numbers.

We want to work with complex numbers.

Ex: Simplify each radical.

① $\sqrt{-25} = \sqrt{-1 \cdot 25} = \sqrt{-1} \cdot \sqrt{25}$
$= j\,5 = 5j$

② $\sqrt{-49} = 7j$ $\qquad \sqrt{49} = 7 \qquad 7^2 = 49$

$(7j)^2 = 7j \cdot 7j = 7 \cdot 7 \cdot j \cdot j = 49(-1)$
$= -49$

③ $\sqrt{-48} = \sqrt{-1 \cdot 16 \cdot 3} = \sqrt{-1} \cdot \sqrt{16} \cdot \sqrt{3}$
$= j \cdot 4\sqrt{3}$
$= 4\sqrt{3}\,j = 4j\sqrt{3}$

$\sqrt{-48} = \sqrt{-1 \cdot 4 \cdot 12} = j \cdot 2\sqrt{12}$
$= j \cdot 2\sqrt{4 \cdot 3}$
$= j\,2 \cdot 2\sqrt{3}$
$= 4\sqrt{3}\,j$

Addition and Subtraction

Ex: $(8 - 3j) + (-6 + 5j)$ $\qquad$ Combine like terms
$= 8 - 3j - 6 + 5j = 2 + 2j$

$$(8-3j)-(-6+5j) = 8-3j+6-5j$$
$$= 14-8j$$

Multiplication

$$(8-3j)(-6+5j) \qquad \text{FOIL} \qquad j^2=-1$$

$$= -48+40j+18j-15j^2 \qquad j^2=-1$$
$$= -48+40j+18j+15$$
$$= -33+58j$$

Complex Conjugates

To find the conjugate of a complex number, change the sign of the j term.

Complex Number	Conjugate
$2-3j$	$2+3j$
$-4+j$	$-4-j$
$6+8j$	$6-8j$

$$(a+bj)(a-bj) = a^2-abj+abj-b^2j^2$$
$$= a^2+b^2 \qquad j^2=-1$$

$$(-3+2j)(-3-2j) = (-3)^2+2^2 = 9+4 = 13$$

$$(8-5j)(8+5j) = 89$$

Division

$$\frac{8-3j}{-6+5j}$$

To divide one complex number by another, multiply the numerator and denominator by the conjugate of the denominator.

$$\frac{8-3j}{-6+5j} = \frac{(8-3j)(-6-5j)}{(-6+5j)(-6-5j)}$$

$(a+bj)(a-bj)$
$= a^2+b^2$

$$= \frac{-48-40j+18j+15j^2}{(-6)^2+5^2}$$

$j^2=-1$

$$= \frac{-48-22j-15}{61} = \frac{-63-22j}{61}$$

$$= -\frac{63}{61} - \frac{22}{61}j$$

$$(a+bj)(a-bj) = a^2+b^2$$

$$\frac{-5+2j}{3-4j} = \frac{(-5+2j)(3+4j)}{(3-4j)(3+4j)}$$

$8j^2 = -8$

$$= \frac{-15-20j+6j-8}{3^2+4^2}$$

$$= \frac{-23-14j}{25} = -\frac{23}{25} - \frac{14}{25}j$$

Ex: Solve $2x^2+3=4x$.

$$2x^2-4x+3=0$$

$a=2 \quad b=-4 \quad c=3$

$$x=\frac{4\pm\sqrt{16-24}}{4}=\frac{4\pm\sqrt{-8}}{4}$$

$$=\frac{4\pm\sqrt{-1\cdot 4\cdot 2}}{4}=\frac{4\pm 2\sqrt{2}\,j}{4}$$

$$=\frac{4}{4}\pm\frac{2\sqrt{2}}{4}j=1\pm\frac{\sqrt{2}}{2}j$$

Check: $2\left(1+\frac{\sqrt{2}}{2}j\right)^2-4\left(1+\frac{\sqrt{2}}{2}j\right)+3=0$

HW 15: Monday