

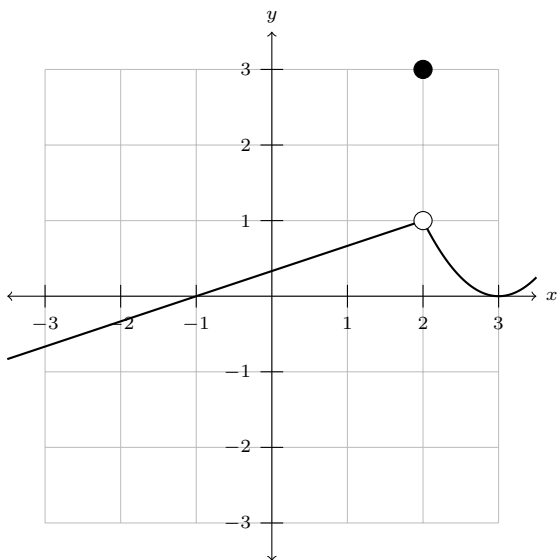
Technical Calculus I

Instructions

- Each assignment is to be done on regular-sized notebook paper.
- Your name and the assignment number should appear at the top of the first sheet.
- Please do not use a red pen or red pencil to do the homework.
- All relevant work is **required**. Failure to show relevant work will result in a large point deduction.

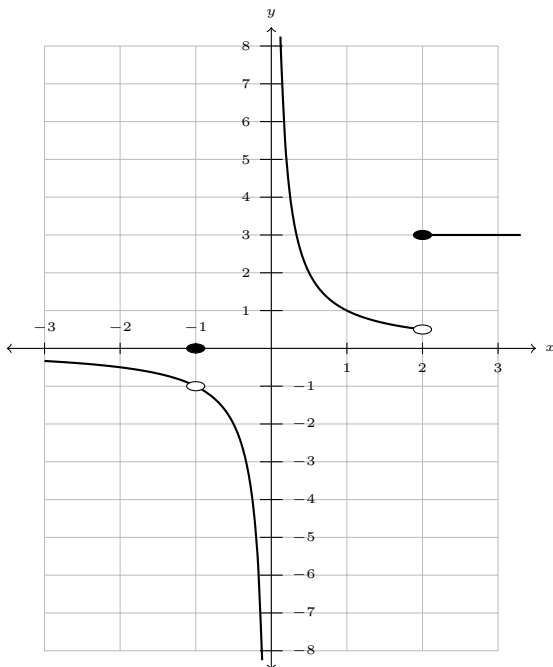
Graded Homework 1 Problems

1. The graph of $y = f(x)$ is shown below.



- A. Find $\lim_{x \rightarrow 2} f(x)$ if it exists.
- B. Find $f(2)$ if it exists.
- C. Find $\lim_{x \rightarrow -1} f(x)$ if it exists.
- D. Find $f(-1)$ if it exists.

2. The graph of $y = f(x)$ is shown below.



- A. Find $\lim_{x \rightarrow -1} f(x)$ if it exists.
 - B. Find $f(-1)$ if it exists.
 - C. Find $\lim_{x \rightarrow 0} f(x)$ if it exists.
 - D. Find $f(0)$ if it exists.
 - E. Find $\lim_{x \rightarrow 2} f(x)$ if it exists.
 - F. Find $f(2)$ if it exists.
3. Let $y = f(x) = \frac{\sin(x)}{x}$. Copy the following table, fill it out, and use it to determine $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$. (Make sure your calculator is in **radian mode** when evaluating $\sin(x)$. In calculus, angles must always be in radians for computations.)

x	y	x	y
1		-1	
0.5		-0.5	
0.2		-0.2	
0.1		-0.1	
0.01		-0.01	

4. Let $y = f(x) = \frac{2x^2 - 7x + 6}{x - 2}$. Copy the following table, fill it out, and use it to determine $\lim_{x \rightarrow 2} \frac{2x^2 - 7x + 6}{x - 2}$.

x	y	x	y
2.1		1.9	
2.01		1.99	
2.001		1.999	
2.0001		1.9999	

5. Compute $\lim_{x \rightarrow -3} (2x^3 - x^2 + 3x - 2)$.
6. Compute $\lim_{x \rightarrow \sqrt{2}} (x^2 - 2)(5x^4 - 3x^2 + 1)$.
7. Compute $\lim_{x \rightarrow 2/3} \frac{4x^2 + 7}{2x^2 - 8x + 1}$.
8. Compute $\lim_{x \rightarrow 0} \frac{13x^4 - 10x^3 + 8x^2}{5x^2}$.
9. Compute $\lim_{x \rightarrow -2} \frac{x + 2}{x^3 + 8}$.
10. Compute $\lim_{h \rightarrow 0} \frac{(2 + h)^2 - 4}{h}$.
11. Compute $\lim_{h \rightarrow 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h}$.

Graded Homework 2 Problems

1. **A.** Find the slope of the line tangent to the graph of $f(x) = x^2 + 1$ when $x = 2$.
B. Find the slope-intercept form of the tangent line.
C. Sketch a graph which includes both the function and the tangent line. Be sure to include the point the two have in common.
2. **A.** Find the slope of the line tangent to the graph of $g(x) = 3x - 5$ when $x = 4$.
B. Find the slope-intercept form of the tangent line.
C. Are there any differences between $g(x)$ and the tangent line? Explain your answer.
3. **A.** Find the slope of the line tangent to the graph of $h(x) = x^3 - x$ when $x = 0$.
B. Find the slope-intercept form of the tangent line.
4. **A.** Find the slope of the line tangent to the graph of $f(x) = x^2 + 4x + 7$ for any value of x .
B. Find the slope of the tangent line when $x = -3$.
C. Find the slope-intercept form of the tangent line.
5. **A.** Find the slope of the line tangent to the graph of $h(x) = x^3$ for any value of x .
B. Find the slope of the tangent line when $x = 1/2$.
C. Find the slope-intercept form of the tangent line.
6. **A.** Find the slope of the line tangent to the graph of $g(x) = \frac{1}{2x}$ for any value of x .
B. Find the slope of the tangent line when $x = 1/4$.
C. Find the slope-intercept form of the tangent line.

Graded Homework 3 Problems

In the following problems, $s = f(t)$ is the position s at time t of an object in linear motion.

1. Suppose $s = 4t + 1$.
 - A. Find \bar{v} from $t = 0$ to $t = 3$.
 - B. Find v for any time t .
 - C. What does part B say about \bar{v} ?
2. Suppose $s = t^2 + 3t$.
 - A. Find \bar{v} from $t = 2$ to $t = 3$.
 - B. Find v for any time t .
 - C. What is v when $t = 2$?
 - D. What is v when $t = 3$?
 - E. Why is the \bar{v} found in part A the average of the v found in part C and the v found in part D?
3. Suppose $s = \frac{1}{t}$.
 - A. What is s when $t = 1$?
 - B. What is s when $t = 2$?
 - C. What is \bar{v} from $t = 1$ to $t = 2$?
 - D. Why is the above \bar{v} negative?
 - E. Compute v for any time t .
 - F. Why is v negative when $t > 0$?

Graded Homework 4 Problems

1. Suppose $y = x^2 - 1$.
 - A. Use the definition of the derivative to compute $\frac{dy}{dx}$.
 - B. Find the slope of the line tangent to the graph of y when $x = -1$.
 - C. Find the slope-intercept form of the tangent line.
2.
 - A. Use the definition of the derivative to compute $f'(x)$ if $f(x) = \sqrt{x-1}$.
 - B. What is $f'(5)$?
3. Suppose $y = 4x - 3$. Explain why y' is a constant without computing it.
4. If $s = 4t^2 - 2t + 1$ is the position s at time t of an object in linear motion, then $v = s' = 8t - 2$ and $v' = 8$. What does v' represent? Explain your answer.

Graded Homework 5 Problems

1. Find y' if $y = x^7$.
2. Find $\frac{dy}{dx}$ if $y = -13x^3$.
3. Find $f'(x)$ if $f(x) = 3x - \frac{4}{5}x^5$.
4. Find $g'(x)$ if $g(x) = \frac{8}{3}x^4 - \frac{x^3}{2} + 2\sqrt{5}x - \pi$.
5. Find $\left. \frac{dy}{dx} \right|_{x=-1}$ if $y = (2x^5 - 2)(3x^2 + x)$.
6. Find $y'(0)$ if $y = (x^2 - 3x + 1)(-2x^3 - 8)$.
7. Find $h'(1)$ if $h(x) = (\sqrt{5}x - 2\sqrt{5})^2$.
8. Find y' if $y = (x - 2)^3$.
9. Find $\frac{dy}{dt}$ if $y = 7t - (t - 7)^2$.
10. Find $f'(z)$ if $f(z) = \frac{16z^3 - 4z^2 + 3z - 1}{7}$.
11. Find $\frac{dx}{dy}$ if $x = 13(5 - y^{20}) + 6y^{40}$.
12. **A.** Find the slope of the tangent line of $y = -3x^2 + 8x - 5$ when $x = -2$.
B. Find the slope-intercept form of the tangent line.
13. Let $s = 2t^3 + 6t^2 - 1$ be the position of an object in linear motion. Find v when $t = 2$.

Graded Homework 6 Problems

1. Use the product rule to find y' if $y = (x + 1)(3x - 4)$. Simplify the answer.
2. Use the product rule to find $\frac{dy}{dx}$ if $y = (2x - 5)(x^2 + 4)$. Simplify the answer.
3. Use the product rule to find $f'(x)$ if $f(x) = x^2(x^4 - 2x^2 + 1)$. Simplify the answer.
4. Use the product rule to find y' if $y = (3x^2 - 4)^2$. Simplify the answer.
5. Use the product rule to find $\frac{dy}{dx}$ if $y = x(x + 1)(x + 2)$. Simplify the answer.
6. Use the quotient rule to find y' if $y = \frac{2}{3x}$. Simplify the answer.
7. Use the quotient rule to find $\frac{dy}{dx}$ if $y = \frac{x}{2x + 3}$. Simplify the answer.
8. Use the quotient rule to find $f'(x)$ if $f(x) = \frac{x - 1}{3x^2 - 2x - 1}$. Simplify the answer.

9. **A.** Find the slope of the tangent line of $y = \frac{2x - 4}{x + 2}$ when $x = 0$.
B. Find the slope-intercept form of the tangent line.
10. If $s = \frac{1}{t + 1}$ is the position of an object in linear motion, find v when $t = 8$.

Graded Homework 7 Problems

1. Use the general power rule to find y' in each of the following parts.
- A.** $y = (8x - 1)^4$
B. $y = 7(x^2 + 8)^3$
C. $y = 4\sqrt{3x^2 + 2}$
D. $y = \frac{1}{2x - 3}$
E. $y = \frac{-2}{(3x - 4)^2}$
F. $y = \frac{3}{\sqrt{x^2 + 1}}$
2. Find $f'(x)$ in each of the following parts.
- A.** $f(x) = 3x^2(5x + 2)^3$
B. $f(x) = \frac{x^2}{(3x - 2)^2}$
3. Suppose $s = (2t + 1)^3$ is the position at time t of an object in linear motion. Find v when $t = 0$.
4. Find the slope of the line tangent to the graph of $f(x) = 3(1 - x)^5$ when $x = 0$.

Graded Homework 8 Problems

1. Use implicit differentiation to find $\frac{dy}{dx}$ in each of the following parts.
- A.** $2x^2 - 5y^2 = 1$
B. $x^3 + 3xy = 8$
C. $3x^2 + xy - 4y^2 = 2$
D. $\frac{4x}{y^2} = 1$
2. The graph of $x^2 + y^2 = 5$ is a circle with center $(0, 0)$ and radius $\sqrt{5}$.
- A.** What two points are on the graph of $x^2 + y^2 = 5$ when $y = 1$?
B. Find the slopes of the lines tangent to the graph of $x^2 + y^2 = 5$ when $y = 1$.
C. Find the slope-intercept forms of the lines tangent to the graph of $x^2 + y^2 = 5$ when $y = 1$.
D. Sketch a graph which shows the circle and the two tangent lines.

Graded Homework 9 Problems

1. Find $f'''(x)$ if $f(x) = 4x^5 - 2x^2 + 3x - 5$.
2. Find $y^{(4)}$ if $y = \frac{-3}{x}$.
3. Find $\frac{d^3y}{dx^3}$ if $y = \frac{x+1}{x-2}$.
4. Find $g''(x)$ if $g(x) = \sqrt{4-x^2}$.
5. Suppose that $s = 3t^3 - 2t^2 + 7t - 1$ is the position at time t of an object in linear motion. Find a when $t = 5$.

Graded Homework 10 Problems

Complete each part for the functions $y = f(x)$ listed below.

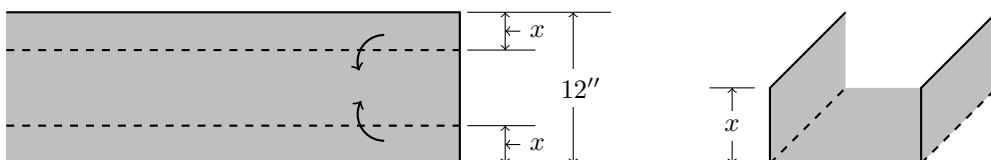
- A. Find the all intercepts of the graph of $y = f(x)$.
- B. Find all critical points.
- C. Find the possible inflection points.
- D. Determine the intervals on which y is increasing, decreasing, concave up, and concave down.
- E. Which points are relative maximums? Which points are relative minimums? Which points are inflection points?
- F. Sketch a graph of y . Include all intercepts, relative extrema, and inflection points.

Problems

1. $y = 6x^2 + x - 15$ **Note:** $y = (3x+5)(2x-3)$, $y' = 12x+1$, and $y'' = 12$.
2. $y = 2x^3 + 8x^2$ **Note:** $y = 2x^2(x+4)$, $y' = 2x(3x+8)$, and $y'' = 4(3x+4)$.
3. $y = x^4 - 3x^3 + 3x^2 - x$ **Note:** $y = x(x-1)^3$, $y' = (x-1)^2(4x-1)$ and $y'' = 6(x-1)(2x-1)$.
4. $y = 25(x^2-1)^3$ **Note:** $y = 25(x-1)^3(x+1)^3$, $y' = 150x(x-1)^2(x+1)^2$, and $y'' = 150(x-1)(x+1)(5x^2-1)$.
5. $y = x^7 + x^6$ **Note:** $y = x^6(x+1)$, $y' = x^5(7x+6)$, and $y'' = 6x^4(7x+5)$.

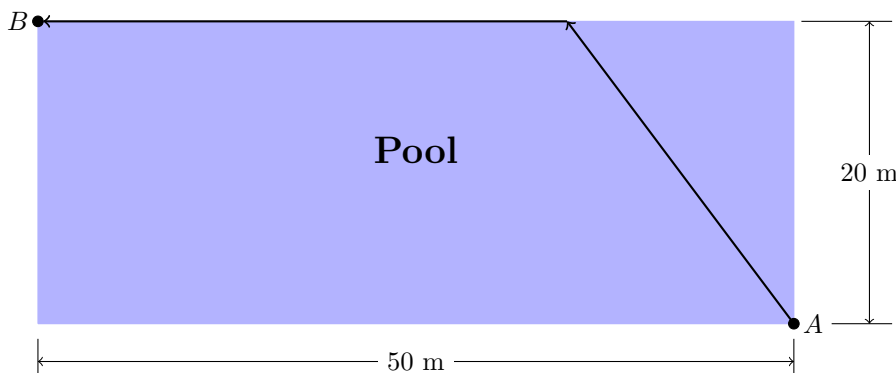
Graded Homework 11 Problems

1. Find two positive numbers such that their sum is 40 and their product is as large as possible.
2. Find the dimensions of the rectangle which has area 9 and the smallest possible perimeter.
3. A long rectangular sheet of aluminum that is 12'' wide is to be made into a gutter as shown in the following diagram.



Determine the value of x such that the gutter can hold the largest possible volume of water.

4. Find the line which is tangent to $y = -x^3 + 6x^2 + 4x - 4$ and which has the largest possible slope.
5. Let $s(t) = -t^3 + 3t^2 + 24t$ be the location function of an object in linear motion when $0 \leq t \leq 4$. When does the object attain its maximum velocity? What is the maximum velocity of the object? What is the location of the object when it attains its maximum velocity? What is the acceleration of the object when it attains its maximum velocity?
6. Z can swim at a rate of 1 m/s and run at a rate of 2 m/s. Z plans to move from point A to point B by first swimming in a straight line across the pool and then running along the edge of the pool. If the quickest route is taken, how long will Z take?



Graded Homework 12 Problems

1. Find y' in each of the following parts.
 - A. $y = 3 \sin(6x)$
 - B. $y = \sin(4x - 3)$
 - C. $y = 6 \sin(x^2 - 2)$
 - D. $y = 2 \cos(3x^2)$
 - E. $y = 7 \cos(x^2 + 2x + 1)$

- F. $y = \cos((x+2)^3)$
- G. $y = 4 \sin^2(2x^2)$
- H. $y = \sin(9x) \cos(5x)$
- I. $y = \cos(\sin(2x))$
- J. $y = -4 \tan(\sqrt{x})$
- K. $y = \sec(2-8x)$
- L. $y = -\frac{\cot(1-2x)}{2}$
- M. $y = -\csc(4x^2)$
- N. $y = \tan(x^2-1)$
- O. $y = 4 \sec(x^3)$
- P. $y = \cot(4x^5)$
- Q. $y = -8 \csc(\sin(x))$
- R. $y = \sec^2(2x+4)$
- S. $y = 7 \tan((3x^2+1)^4)$
- T. $y = 2 \sin^2(3x) + 2 \cos^2(3x)$

2. Find the slope-intercept form of the tangent line of $y = \sqrt{2} \cos(3x)$ when $x = \frac{\pi}{12}$.

3. Find y' in each of the following parts.

- A. $y = 4 \sin^{-1}(2x)$
- B. $y = \cos^{-1}(3x^2)$
- C. $y = 2 \tan^{-1}(x^2-1)$
- D. $y = 2 \sin^{-1}(3x) - 4 \cos^{-1}(3x)$
- E. $y = \tan^{-1}\left(\frac{3}{x^2}\right)$

4. Find the slope-intercept form of the tangent line of $y = 12 \tan^{-1}\left(\frac{x}{3}\right)$ when $x = -3$.

5. Find y' in each of the following parts.

- A. $y = \ln(3x)$
- B. $y = 5 \ln(2x-3)$
- C. $y = \ln(x^3-x^2+6)$
- D. $y = \ln((x^2-1)^3)$
- E. $y = \ln(\cos(2x))$
- F. $y = -4 \ln\left(\frac{5}{x}\right)$
- G. $y = \sin(\ln(x))$

6. Use logarithmic differentiation to find $\frac{dy}{dx}$.

A. $y = x^{3x^2}$

B. $y = \frac{x^4(2x-3)^2}{(x+1)^3}$

7. Find y' in each of the following parts.

A. $y = e^{4x^3}$

B. $y = e^{1/x}$

C. $y = \frac{2e^{7x}}{4e^{-5x}}$

D. $y = \sqrt{9e^x}$

E. $y = 3e^{\sin(x)}$

F. $y = \cos(e^{2x})$

G. $y = 6 \ln(e^{-2x})$

H. $y = e^{3 \ln(4x)}$

Graded Homework 13 Problems

Determine each of the following indefinite integrals.

1. $\int 3x^7 dx$

2. $\int \frac{2}{\sqrt{x}} dx$

3. $\int \frac{4}{3x^2} dx$

4. $\int \frac{x^4}{4} dx$

5. $\int \frac{x^2}{3x^{3/2}} dx$

6. $\int 3x^4 - 2x^3 + x - 8 dx$

7. $\int (2x+1)(x-2) dx$

8. $\int x(x+3)^2 dx$

9. $\int \frac{1}{x^2} - \frac{3}{x^4} + (\sqrt{x})^5 dx$

10. $\int \frac{(2x^2+x)^2}{x^2} dx$

Graded Homework 14 Problems

1. Find $y = f(x)$ if $y' = \frac{3}{x^2}$ and the graph of y passes through the point $(1, -4)$.
2. Find $y = f(x)$ if $y' = 12x^2 - 6x + 4$ and the graph of y passes through the point $(-2, -48)$.
3. Find the position function s of an object in linear motion if $a = 3t + 2$, $v(0) = -2$, and $s(0) = 1/2$.
4. How long does it take for an object dropped from a height of 1000 meters to hit the ground? Assume air resistance is negligible.
5. An object is thrown straight up into the air from a height of 3 meters with an initial velocity of 20 m/s. What is the maximum height of the object? When does the object hit the ground?
6. What is the initial velocity of an object that reaches a maximum height of 50 meters when it is thrown straight up into the air from the ground?
7. An object is thrown straight up into the air from a height of one meter. The object hits the ground 3 seconds after it was thrown. What was the maximum height of the object?

Graded Homework 15 Problems

1. Evaluate $\int_1^4 x^3 - \frac{2}{x^2} + 5 \, dx$.
2. Evaluate $\int_{-3}^{-1} -3x^2 + 4x - 1 \, dx$.
3. Evaluate $\int_{-3}^3 \frac{x^3}{3} - \frac{x}{2} \, dx$.
4. Find the area of the region bounded by the curves $y = \frac{1}{x^2}$, $x = 1$, $x = 4$, and $y = 0$. A sketch of the region is required.
5. Find the area of the region bounded by the curves $y = x^3 + 1$, $x = 1$, and $y = 0$. A sketch of the region is required.
6. Find the area of the region bounded by the curves $y = \sqrt{x}$, $y = 2 - x^2$, and $y = 0$ when $x > 0$. A sketch of the region is required.

Graded Homework 16 Problems

1. Evaluate $\int (3x - 5)^6 \, dx$.
2. Evaluate $\int \frac{1}{(8 - 7x)^3} \, dx$.
3. Evaluate $\int \sqrt{2x - 3} \, dx$.
4. Evaluate $\int \frac{1}{\sqrt{4 - 3x}} \, dx$.

5. Evaluate $\int 4x^2 (x^3 - 2)^4 dx$.
6. Evaluate $\int \frac{x}{7(4x^2 - 5)^2} dx$.
7. Evaluate $\int (x^2 - 1)(2x^3 - 6x + 1)^5 dx$.
8. Evaluate $\int \frac{x - 1}{(x^2 - 2x - 4)^4} dx$.
9. Evaluate $\int \frac{(3 - 2 \ln(x))^2}{2x} dx$.
10. Evaluate $\int \frac{1}{2x(\ln(2x))^2} dx$.
11. Evaluate $\int \cos(4x)(\sin(4x) + 1)^3 dx$.
12. Evaluate $\int \sec^4(5x) \tan(5x) dx$.
13. Evaluate $\int \tan(x + 1) \sec^2(x + 1) dx$.
14. Evaluate $\int \frac{\tan^{-1}(6x)}{1 + 36x^2} dx$.
15. Evaluate $\int \frac{x(1 - e^{-4x^2})^3}{e^{4x^2}} dx$.
16. Evaluate $\int \frac{4}{4x^2 - 20x + 25} dx$.
17. Evaluate $\int (x^3 - 6x^2 + 12x - 8)^7 dx$.
18. Evaluate $\int \frac{x}{(2x - 1)^4} dx$.
19. Evaluate $\int x\sqrt{1 - x} dx$.
20. Evaluate $\int \frac{x^2}{\sqrt{3x - 4}} dx$.

Graded Homework 17 Problems

1. Evaluate $\int \frac{5}{3x} dx$.
2. Evaluate $\int \frac{x^3}{3x^4 - 7} dx$.
3. Evaluate $\int \frac{\cos(2x)}{3 \sin(2x) + 1} dx$.
4. Evaluate $\int \frac{e^{5x}}{5 - 4e^{5x}} dx$.
5. Evaluate $\int 6e^{3x} dx$.
6. Evaluate $\int \frac{1}{e^{2x}} dx$.
7. Evaluate $\int x^3 e^{-4x^4} dx$.
8. Evaluate $\int \cos(7x) dx$.
9. Evaluate $\int \sin\left(\frac{x}{2}\right) dx$.
10. Evaluate $\int x \sec(x^2) dx$.
11. Evaluate $\int 14 \sec(3x) \tan(3x) dx$.
12. Evaluate $\int e^{3x} \sin(4e^{3x}) dx$.
13. Evaluate $\int \cos(\tan(2x)) \sec^2(2x) dx$.
14. Evaluate $\int \sqrt{x} \tan(x^{3/2}) dx$.
15. Evaluate $\int \frac{\sec(\ln(x^2))}{x} dx$.