

Technical Calculus II

Instructions

- Each assignment is to be done on regular-sized notebook paper.
- Your name and the assignment number should appear at the top of the first sheet.
- Please do not use a red pen or red pencil to do the homework.
- All relevant work is **required**. Failure to show relevant work will result in a large point deduction.

Graded Homework 1 Problems

1. **A.** Sketch the region bounded by the curves $y = 2x + 1$, $x = 1$, $x = 2$, and $y = 0$.
B. Find the area of this region.
2. **A.** Sketch the region bounded by the curves $3x + 2y = 6$, $x = 0$, and $y = 0$.
B. Find the area of this region.
3. **A.** Sketch the region bounded by the curves $y = x^2 - 4$ and $y = 0$.
B. Find the area of this region.
4. **A.** Sketch the region bounded by the curves $y = \sqrt{2 - x}$ and $x + y = 2$.
B. Find the area of this region.
5. **A.** Sketch the region bounded by the curves $x = \frac{1}{y}$, $x = 0$, $y = 1$, and $y = 4$.
B. Find the area of this region.
6. **A.** Sketch the region bounded by the curves $y = 2 \cos\left(\frac{x}{2}\right)$, $y = 0$, $x = \frac{\pi}{2}$, and $x = \frac{5\pi}{4}$.
B. Find the area of this region.
7. **A.** Sketch the region bounded by the curves $y = \cos(x)$, $y = \sin(x)$, $x = -\frac{3\pi}{4}$, and $x = \frac{\pi}{4}$.
B. Find the area of this region.
8. **A.** Sketch the region bounded by the curves $x = y^3$ and $x = y$.
B. Find the area of this region.

Graded Homework 2 Problems

1. **A.** Sketch the solid generated when the region bounded by the curves $y = 3x + 2$, $x = 0$, $x = 2$, and $y = 0$ is rotated around the x -axis.
B. Find the volume of this solid.
2. **A.** Sketch the solid generated when the region bounded by the curves $y = x^3$, $y = 0$, $x = -1$, and $x = -2$ is rotated around the x -axis.
B. Find the volume of this solid.
3. **A.** Sketch the solid generated when the region bounded by the curves $3x + 2y = 13$, $y = 2$, $y = 5$, and $x = 0$ is rotated around the y -axis.

- B.** Find the volume of this solid.
4. **A.** Sketch the solid generated when the region bounded by the curves $y = \frac{1}{x+1}$, $y = \frac{1}{2}$, and $x = 0$ is rotated around the y -axis.
B. Find the volume of this solid.
5. **A.** Sketch the solid generated when the region bounded by the curves $y = \sqrt{x+1}$, $x = 0$, and $y = 3$ is rotated around the x -axis.
B. Find the volume of this solid.
6. **A.** Sketch the solid generated when the region bounded by the curves $y = x^4$ and $y = 15x - 14$ is rotated around the x -axis.
B. Find the volume of this solid.
7. **A.** Sketch the solid generated when the region bounded by the curves $y = \sqrt{5-x}$, $x = 1$, and $y = 1$ is rotated around the y -axis.
B. Find the volume of this solid.

Graded Homework 3 Problems

1. **A.** Sketch the solid generated when the region bounded by the curves $y = x^2$, $x = 4$, and $y = 0$ is rotated around the y -axis.
B. Find the volume of this solid using the shell method.
2. **A.** Sketch the solid generated when the region bounded by the curves $y = \frac{1}{x}$, $y = 4x$, and $x = 1$ is rotated around the y -axis.
B. Find the volume of this solid using the shell method.
3. **A.** Sketch the solid generated when the region bounded by the curves $x = 2y - y^2$ and $x = 0$ is rotated around the x -axis.
B. Find the volume of this solid using the shell method.

Graded Homework 4 Problems

1. **A.** Sketch the the region bounded by the curves $y = 3x + 1$, $y = 4x$, and $x = 0$.
B. Find the center of mass of this region.
2. **A.** Sketch the the region bounded by the curves $y = 4 - x^2$ and $y = 0$.
B. Find the center of mass of this region.
3. **A.** Sketch the solid generated when the the region bounded by the curves $y = 3x + 1$, $y = 4x$, and $x = 0$ is rotated around the x -axis.
B. Find the center of mass of this solid.
4. **A.** Sketch the solid generated when the the region bounded by the curves $y = \sqrt{x-1}$, $y = 0$, and $x = 5$ is rotated around the x -axis.
B. Find the center of mass of this solid.

Graded Homework 5 Problems

Use partial fractions to evaluate each of the following integrals.

1. $\int \frac{x+9}{x^2-2x-3} dx$

2. $\int \frac{16x+28}{6x^2-13x-5} dx$

3. $\int \frac{3x^2+5x-4}{x^3+x^2-x-1} dx$

4. $\int \frac{x^4+2x^3-3x^2-x-2}{x^2-4} dx$

Graded Homework 6 Problems

Use integration by parts to evaluate each of the following integrals.

1. $\int \ln(x) dx$

2. $\int x^4 \ln(3x) dx$

3. $\int \frac{\ln(2x)}{x^2} dx$

4. $\int x \sin(x) dx$

5. $\int x^2 e^x dx$

6. $\int x \sec^2(x) dx$

7. $\int e^x \sin(x) dx$

8. $\int \frac{x}{\sqrt{2x+1}} dx$

9. $\int \ln(x^2-1) dx$

10. $\int_0^1 \ln(x+1) dx$

Graded Homework 7 Problems

Use trigonometric substitution to evaluate each of the following integrals.

1. $\int \frac{1}{\sqrt{16-9x^2}} dx$

2. $\int \frac{1}{\sqrt{16x^2+1}} dx$

3. $\int \frac{1}{\sqrt{x^2-9}} dx$

4. $\int \frac{1}{3x\sqrt{9x^2+16}} dx$

5. $\int_{-5}^5 \sqrt{25-x^2} dx$

Hint: You will need to use the following identity: $\cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta))$

6. $\int \sqrt{x^2-1} dx$

Graded Homework 8 Problems

Use the table of integrals to evaluate each integral.

1. $\int \sqrt{9-x^2} dx$

2. $\int \frac{1}{\sqrt{2-4x^2}} dx$

3. $\int \frac{\sqrt{4-16x^2}}{x^2} dx$

4. $\int \frac{1}{2x\sqrt{3-3x^2}} dx$

5. $\int \frac{1}{4x^2\sqrt{7-5x^2}} dx$

Graded Homework 9 Problems

1. Solve $\frac{dy}{dx} = x^2y^2 + y^2$.

2. Solve $x^2dy = y^3dx$ if $y(1) = \frac{1}{2}$.

3. Solve $\csc(x) dy - \sec(y) dx = 0$.

4. Solve $(x^3+1) dy - x^2dx = 0$ if $y(0) = \frac{3}{2}$.

- Solve $\frac{dy}{dx} = e^{2x} - y$.
- Solve $\cos(x) dy = \tan(x) \cos^2(2y) dx$ if $y(0) = \frac{\pi}{8}$.

Graded Homework 10 Problems

- Solve $x dy = x^3 dx - y dx$.
- Solve $y dx - x dy - x^6 dx = 0$.
- Solve $x dy + x^3 y^2 dx = y dx + xy^2 dx$.
- Solve $xy dy = x^4 dx + y^2 dx$.
- Solve $x^2 y^3 dx = y^5 dy - x^3 y^2 dy$.
- Solve $y dx = 3 dx - x dy$ if $y(1) = 4$.

Graded Homework 11 Problems

- Solve $y' + xy = 0$.
- Solve $y' + 4y = 7$ if $y(0) = 2$.
- Solve $y' - 2xy = 4x$ if $y(0) = -3$.
- Solve $y' + 3x^2 y = 6x^5$.
- Solve $xy' + y = x e^{x^2}$.
- Solve $y' + \sin(x) y = \sin(x)$.

Graded Homework 12 Problems

- Find the equation of the curve $y = f(x)$ which passes through $(-1, -3)$ and has the following slope.

$$\frac{dy}{dx} = \frac{3x^2 - 2x}{2y}$$

- Find the equation of the curve $y = f(x)$ which passes through $(0, 2)$ and has the following slope.

$$\frac{dy}{dx} = 7 - 4y$$

- Find the position function s of an object in linear motion if $a = 3t + 2$, $v(0) = -2$, and $s(0) = 1/2$.
- How long does it take for an object dropped from a height of 1000 meters to hit the ground? Assume air resistance is negligible.
- An object is thrown straight up into the air from a height of 3 meters with an initial velocity of 20 m/sec. What is the maximum height of the object? When does the object hit the ground?
- What is the initial velocity of an object that reaches a maximum height of 50 meters when it is thrown straight up into the air from the ground?

7. An object is thrown straight up into the air from a height of one meter. The object hits the ground 3 seconds after it was thrown. What was the maximum height of the object?
8. The half-life of Neptunium-237 is 2.25 million years. How long will it take a pure one gram sample of Neptunium-237 to decay to a substance of which only 0.01% is Neptunium-237?

Graded Homework 13 Problems

Solve each of the following differential equations.

1. $y'' - 3y' - 10y = 0$.
2. $y'' - y' = 12y$, $y(0) = 9$, $y'(0) = 8$.
3. $4y'' + 12y' - 7y = 0$
4. $6y'' - y' - 12y = 0$, $y(0) = 2$, $y'(0) = 3$
5. $y'' - 12y' + 36y = 0$
6. $49y'' - 112y' + 64y = 0$
7. $y'' - 2y' + 10y = 0$
8. $y'' + 25y = 0$, $y(0) = 3$, $y'(0) = 10$
9. $y'' - 10y' + 34y = 0$, $y(0) = 1$, $y'(0) = -1$

Graded Homework 14 Problems

1. Determine $\mathcal{L} [3e^{-t} - 2e^{4t}]$.
2. Determine $\mathcal{L} [-2e^{3t} + 5te^{-4t}]$.
3. Determine $\mathcal{L} [e^{7t} \cos(6t) - 2e^{-2t} \sin(t)]$.
4. Determine $\mathcal{L} [t^3 - t^2 + 2t - 3]$.
5. Determine $\mathcal{L} [y' - 3y]$ in terms of s and $\mathcal{L}[y]$ if $y(0) = 1$.
6. Determine $\mathcal{L} [y'' + 2y' - y]$ in terms of s and $\mathcal{L}[y]$ if $y(0) = 0$ and $y'(0) = 1$.
7. Determine $\mathcal{L} [y'' - 7y' + 8y]$ in terms of s and $\mathcal{L}[y]$ if $y(0) = -2$ and $y'(0) = 3$.
8. Find y if $\mathcal{L}[y] = \frac{3}{s^5}$.
9. Find y if $\mathcal{L}[y] = \frac{2s - 1}{s^2 - 2s + 1}$.
10. Find y if $\mathcal{L}[y] = \frac{6 - 4s}{s^2 + 4}$.
11. Find y if $\mathcal{L}[y] = \frac{5s + 1}{s^2 - 2s + 10}$.

Graded Homework 15 Problems

Use Laplace transforms to solve each of the following differential equations.

1. $y'' + 2y' + y = 0$, $y(0) = 0$, $y'(0) = -2$

2. $y' + 3y = e^{-3t}$, $y(0) = 1$

3. $y'' - 4y' + 5y = 0$, $y(0) = 1$, $y'(0) = 2$

4. $3y'' + 12y = \cos(2t)$, $y(0) = 0$, $y'(0) = 0$

5. $y'' - 2y' + 2y = 0$, $y(0) = 1$, $y'(0) = 1$

Graded Homework 16 Problems

1. Write each of the following series without using sigma notation.

A. $\sum_{n=1}^5 (2n - 1)$

B. $\sum_{n=1}^4 (-1)^n \frac{n+1}{n}$

C. $\sum_{n=1}^6 (n^2 - 1)$

D. $\sum_{n=1}^4 \frac{1}{\sqrt{n}}$

2. Write each of the following series using sigma notation.

A. $2 + 4 + 6 + 8 + 10 + \cdots + 50$

B. $1 + 4 + 9 + 16 + \cdots + 121$

C. $\frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \cdots + \frac{k}{3^k}$

3. Determine if each of the following series converges or diverges.

A. $2 + 4 + 6 + \cdots + 2n + \cdots$

B. $\sum_{n=1}^{\infty} \frac{4n-1}{2n}$

C. $\sum_{n=1}^{\infty} \frac{1}{4^n}$

D. $\sum_{n=1}^{\infty} \frac{1}{n^3}$

E. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

$$\text{F. } \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$$

$$\text{G. } \sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$$

$$\text{H. } \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + 1}}$$

$$\text{I. } \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2 + 1}}$$

$$\text{J. } \sum_{n=1}^{\infty} \frac{1}{2^n + n}$$

$$\text{K. } \sum_{n=1}^{\infty} \frac{n}{2n^2 + 1}$$

$$\text{L. } \sum_{n=1}^{\infty} \frac{2}{n^2 + 2n - 1}$$

$$\text{M. } \sum_{n=1}^{\infty} \frac{n+1}{\sqrt{n}}$$

$$\text{N. } \sum_{n=1}^{\infty} \frac{2^n}{n}$$

$$\text{O. } \sum_{n=1}^{\infty} \frac{1}{n^n}$$

Graded Homework 17 Problems

Use the ratio test on the following series. If the ratio test fails, use another test to determine if the series converges or diverges.

$$1. \sum_{n=1}^{\infty} \frac{2}{n!}$$

$$2. \sum_{n=1}^{\infty} \frac{2^n}{n^2}$$

$$3. \sum_{n=1}^{\infty} \frac{1}{3n}$$

$$4. \sum_{n=1}^{\infty} \frac{2^n}{3^n}$$

$$5. \sum_{n=1}^{\infty} \frac{3n^2}{2n^2 - 1}$$