① $\int \ln(x)\,dx$

$u = \ln(x)$ $\qquad$ $dv = dx$

$\frac{du}{dx} = \frac{1}{x}$ $\qquad$ $v = \int dx = x$

$du = \frac{1}{x}dx$

$$uv - \int v\,du = x\ln(x) - \int x\cdot\frac{1}{x}\,dx$$

$$= x\ln(x) - \int dx$$

$$= x\ln(x) - x + C$$

⑧ $\int \frac{x}{\sqrt{2x+1}}\,dx = \int x\,(2x+1)^{-1/2}\,dx$

$u = x$ $\qquad$ $dv = (2x+1)^{-1/2}\,dx$

$du = dx$ $\qquad$ $v = \int (2x+1)^{-1/2}\,dx$

$w = 2x+1$

$\frac{dw}{dx} = 2$ $\qquad$ $dw = 2dx$

$v = \frac{1}{2}\int (2x+1)^{-1/2}\cdot 2dx$

$= \frac{1}{2}\int w^{-1/2}\,dw$

$= \frac{1}{2}\cdot 2w^{1/2} = (2x+1)^{1/2}$

$\int x(2x+1)^{-1/2}\,dx$

$= x(2x+1)^{1/2} - \int (2x+1)^{1/2}\,dx$

$= x(2x+1)^{1/2} - \frac{1}{2}\int (2x+1)^{1/2}\cdot 2dx$

$= x(2x+1)^{1/2} - \frac{1}{2}\cdot\frac{2}{3}(2x+1)^{3/2} + C$

$= x(2x+1)^{1/2} - \frac{1}{3}(2x+1)^{3/2} + C$

7 $\int e^x \sin(x)\,dx$ $\quad u = \sin(x) \quad dv = e^x\,dx$

$$\frac{du}{dx} = \cos(x) \qquad v = e^x$$

$$du = \cos(x)\,dx$$

$$\int e^x \sin(x)\,dx = e^x \sin(x) - \int e^x \cos(x)\,dx$$

$$\boxed{\begin{array}{ll} u = \cos(x) & dv = e^x dx \\ du = -\sin(x)\,dx & v = e^x \end{array}}$$

$$\int e^x \sin(x)\,dx$$

$$= e^x \sin(x) - \left[e^x \cos(x) - \int e^x(-\sin(x))\,dx \right]$$

$$= e^x \sin(x) - e^x \cos(x) - \int e^x \sin(x)\,dx$$

$$2\int e^x \sin(x)\,dx = e^x \sin(x) - e^x \cos(x)$$

$$\int e^x \sin(x)\,dx = \frac{1}{2} e^x \sin(x) - \frac{1}{2} e^x \cos(x) + C$$

9 $\int \ln(x^2-1)\,dx = \int \ln\big((x-1)(x+1)\big)\,dx$

$$= \int \ln(x-1) + \ln(x+1)\,dx$$

$$= \int \ln(x-1)\,dx + \int \ln(x+1)\,dx$$

$$\left[\begin{array}{l} u = \ln(x-1), \quad dv = dx \\ du = \frac{1}{x-1}\,dx, \quad v = x \end{array} \right| \quad \left. \begin{array}{l} u = \ln(x+1), \quad dv = dx \\ du = \frac{1}{x+1}\,dx \quad v = x \end{array} \right]$$

$$\int \ln(x^2-1)\,dx$$

$$= x\ln(x-1) - \int \frac{x}{x-1}\,dx + x\ln(x+1) - \int \frac{x}{x+1}\,dx$$

$$= x\ln(x-1) - \int \frac{x-1+1}{x-1}\,dx + x\ln(x+1) - \int \frac{x+1-1}{x+1}\,dx$$

$$= x\ln(x-1) - \int \frac{x-1}{x-1} + \frac{1}{x-1}\,dx$$

$$+ x\ln(x+1) - \int \frac{x+1}{x+1} - \frac{1}{x+1}\,dx$$

$$= x\ln(x-1) - \int 1 + \frac{1}{x-1}\,dx$$

$$+ x\ln(x+1) - \int 1 - \frac{1}{x+1}\,dx$$

$$= x\ln(x-1) - \left[x + \ln|x-1|\right]$$

$$+ x\ln(x+1) - \left[x - \ln|x+1|\right] + C$$

$$= x\ln(x-1) - x - \ln|x-1| + x\ln|x+1| - x + \ln|x+1| + C$$

$$= -2x + x\ln(x-1) + x\ln|x+1| - \ln|x-1| + \ln|x+1| + C$$

(2) $$\int \frac{\ln(2x)}{x^2}\,dx$$

$u = \ln(2x)$ $\quad dv = \frac{1}{x^2}\,dx$

$\frac{du}{dx} = \frac{1}{2x}\cdot 2 = \frac{1}{x}$ $\quad = x^{-2}\,dx$

$du = \frac{1}{x}\,dx$ $\quad v = \int x^{-2}\,dx$

$= -\frac{1}{x}$

$$uv - \int v\,du = -\frac{\ln(2x)}{x} - \int -\frac{1}{x}\cdot\frac{1}{x}\,dx$$

$$= -\frac{\ln(2x)}{x} + \int x^{-2}\,dx = -\frac{\ln(2x)}{x} - \frac{1}{x} + C$$

⑤ $\int x^2 e^x \, dx$

$u = e^x \qquad dv = x^2\,dx$

$du = e^x dx \qquad v = \frac{x^3}{3}$

Bad choice

$uv - \int v\,du = \frac{x^3}{3}e^x - \int \frac{x^3}{3} e^x\,dx$

$u = x^2 \qquad dv = e^x\,dx$

$du = 2x\,dx \qquad v = e^x$

$$\int x^2 e^x\,dx = x^2 e^x - \int e^x \cdot 2x\,dx$$

$$= x^2 e^x - 2\int x e^x\,dx$$

$u = x \qquad dv = e^x dx$

$du = dx \qquad v = e^x$

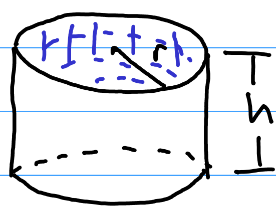
$$= x^2 e^x - 2\left[x e^x - \int e^x\,dx\right]$$

$$= x^2 e^x - 2x e^x + 2\int e^x\,dx$$

$$= x^2 e^x - 2x e^x + 2e^x + C$$

Exam 2

① HW 3: #1, 2 Volumes via shell method



Area of shell $= 2\pi r h$

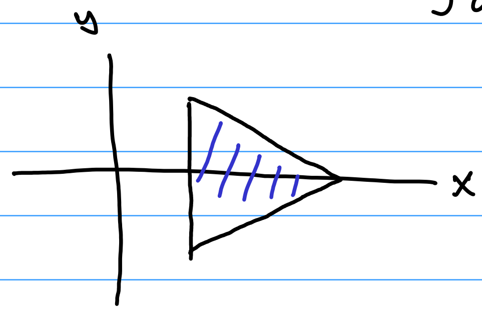
② Center of mass
HW 4

Upper function, Lower function

Region: $m = \int_a^b f(x) - g(x)\,dx$

$M_y = \int_a^b x\,(f(x) - g(x))\,dx$

$M_x = \frac{1}{2}\int_a^b [f(x)]^2 - [g(x)]^2\,dx$



$\bar{y} = 0$

Solid $m = \pi \int_a^b [f(x)]^2 - [g(x)]^2\,dx$

$M_y = \pi \int_a^b x\left[(f(x))^2 - (g(x))^2\right]dx$

③ Partial fractions
HW5: #1,2,3

④ Integration by parts
HW6: #1-6

$\frac{d}{dx}(\ln(u)) = \frac{1}{u}\frac{du}{dx}$

$\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$

$y = e^{8x}$
$y' = 8e^{8x}$

$\int e^u\,du = e^u + C$

$\int e^{-5x}\,dx = -\frac{1}{5}\int e^{-5x}(-5\,dx) = -\frac{1}{5}e^{-5x} + C$